

5.12.1 Quantified Predicate Logic: Truth Tree Problems

A. State which **mistake(s)** are made in each of the following truth trees.

1.

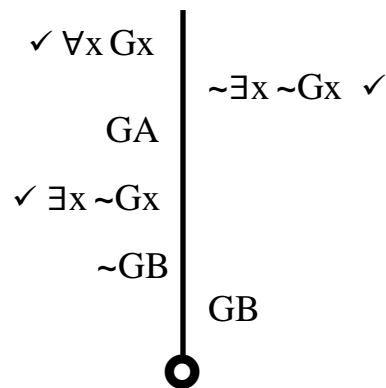
1. Everything is a material object.

\therefore Nothing is an immaterial object.

G: ____ is a material object.

1. $\forall x Gx$

$\therefore \sim \exists x \sim Gx$



Verdict: Invalid

(A, Continued)

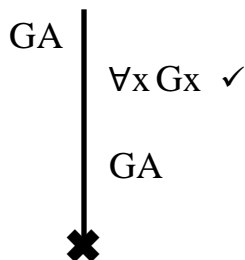
2.

1. The Eiffel Tower is made of metal.

 \therefore Everything is made of metal.

A: The Eiffel Tower G: ____ is made of metal

1. GA

 $\therefore \forall x Gx$ **Verdict:** Valid

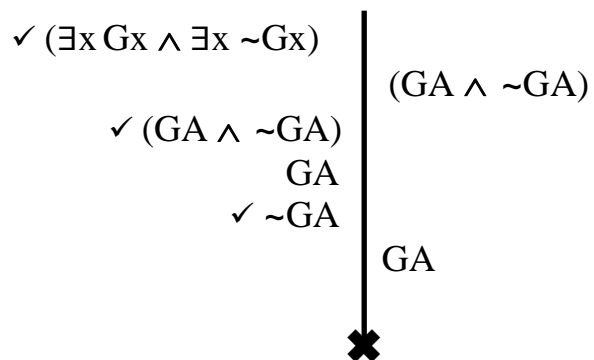
(A, Continued)

3.

1. Something is made of metal, and something isn't.

∴ The Eiffel Tower is made of metal, but it isn't made of metal.

A: The Eiffel Tower **G:** ____ is made of metal

$$1. (\exists x Gx \wedge \exists x \sim Gx)$$
$$\therefore (GA \wedge \sim GA)$$


Verdict: Valid

B. For each of the following formal arguments, build a **truth tree** to decide if the argument is **valid** or **invalid**.

1. $(\forall x Gx \wedge \forall x Hx) \therefore \forall x (Gx \wedge Hx)$
2. $\forall x (Gx \wedge Hx) \therefore (\forall x Gx \wedge \forall x Hx)$
3. $(\exists x Gx \vee \exists x Hx) \therefore \exists x (Gx \vee Hx)$
4. $\forall x Gx \cdot \exists x Hx \therefore \exists x (Gx \wedge Hx)$
5. $\forall x (Gx \vee Hx) \therefore (\forall x Gx \vee \forall x Hx)$
6. $(\forall x Gx \vee \forall x Hx) \therefore \forall x (Gx \vee Hx)$
7. $\sim \forall x (Gx \rightarrow Hx) \therefore \exists x (Gx \wedge Hx)$
8. $\exists x (Gx \wedge Hx) \therefore \sim \forall x (Gx \rightarrow Hx)$
9. $\forall x (Gx \rightarrow Hx) \cdot \exists x (Gx \wedge Hx) \therefore \forall x (Gx \wedge Hx)$

C. Does the following argument come out **valid**, according to the truth tree test? (In particular: beginning with the scope formula “GA,” is **every free occurrence** of the variable ‘x’ replaced by a **new name letter** to yield the conclusion “GA”?)

1. $\exists x GA$
-
- \therefore 2. GA

D. Translate each of the following arguments into formal language; then build a **truth tree** to decide if the argument is **valid or invalid**.

1. Something is a fat cat. \therefore Something is fat, and something is a cat.
2. Something is round, and something is a square. \therefore Something is a round square.
3. All humans are mammals. All mammals are mortals. \therefore All humans are mortals.
4. Some animals are reptiles. Some animals are dogs. \therefore Some reptiles are dogs.
5. Every surfer is either male or female. \therefore Either every surfer is male, or every surfer is female.
6. Every surfer is either male or female. \therefore Either every surfer is male, or some surfer is female.
7. All penguins are animals. No penguins are mammals. \therefore No mammals are animals.
8. No one got a medical degree without going to college. Doctor Zhivago is someone who got a medical degree. \therefore Doctor Zhivago is someone who went to college.
9. Only gamblers are card-counters. Every gambler is either contemplative or talkative. Jake is a talkative gambler. \therefore Jake is a card-counter.
10. All rational animals are mortal. Something is immortal.
 \therefore Something is not an animal.
11. Unexamined lives are not worth living. \therefore If any lives are worth living, examined ones are.

12. Any antique is either attractive or expensive. No tchotchkes are expensive. The Maltese Knick-Knack is an unattractive tchotchke. \therefore The Maltese Knick-Knack is not an antique.

13a. All monoids are semigroups, but not all semigroups are monoids. \therefore It's not the case that: something is a monoid if and only if it's a semigroup.

13b. It's not the case that: something is a monoid if and only if it's a semigroup. \therefore All monoids are semigroups, but not all semigroups are monoids.